# Seismic Imaging Based on Spectral-Element & Adjoint Methods

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# Seismic Wave Propagation ("Forward" Problem) Equation of motion: $\rho \partial_t^2 \mathbf{s} - \nabla \cdot \mathbf{T} = \mathbf{f}$

Boundary condition:

 $\hat{\mathbf{n}} \cdot \mathbf{T} = \mathbf{0}$ 

Initial conditions:

 $\mathbf{s}(\mathbf{x},0) = \mathbf{0}$  $\partial_t \mathbf{s}(\mathbf{x},0) = \mathbf{0}$ 

Earthquake source:

 $\mathbf{f} = -\mathbf{M} \cdot \nabla \delta(\mathbf{x} - \mathbf{x}_{s}) S(t)$ 

# Spectral-Element Method

## Weak form:

$$\int_{\Omega} \rho \, \mathbf{w} \cdot \partial_t^2 \mathbf{s} \, \mathrm{d}^3 \mathbf{x} = - \int_{\Omega} \, \boldsymbol{\nabla} \, \mathbf{w} : \mathbf{T} \, \mathrm{d}^3 \mathbf{x} + \, \mathbf{M} : \boldsymbol{\nabla} \, \mathbf{w}(\mathbf{x}_s) S(t)$$

- implicitly accounts for free-surface boundary condition
- fluid-solid coupling terms may be explicitly added

# Spectral-Element Method

## Finite-elements:

- hexahedral elements
- Gauss-Lobatto-Legrendre quadrature
- diagonal mass matrix
- explicit time-marching scheme

 $M\ddot{U} = -KU + F$ 





## Parallel Implementation

Regional mesh partitioning



## Parallel Implementation

#### Global mesh partitioning





Cubed Sphere: 6 n<sup>2</sup> mesh slices

## Coffee Cup Simulation



#### SPECFEM3D\_SESAME

#### Soon: GPU version

Daniel Peter, Max Riethman, Dimitri Komatitsch, Olaf Schenk, Tarje Nissen-Meyer, Piero Basini, Lapo Boschi





#### Starting 3D Mantle Model: S362ANI







Adjoint source = SP( body-wave measurement) + LP (surface-wave measurement)

$$\delta \chi^{\pi} = \int K_{c} \delta \ln c + K_{\beta_{v}} \delta \ln \beta_{v} + K_{\beta_{h}} \delta \ln \beta_{h} + K_{\eta} \delta \ln \eta + K_{\rho} \delta \ln \rho \, dV$$

#### **Reduction in Misfit: First Iteration**































































































































#### Mediterranean-Calabria Paleotectonics









Wortel & Spakman (2000)













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## Towards Global Adjoint Tomography

 $254 \text{ CMT events} \\ 5.8 \le \text{Mw} \le 7$ 



shallow: d $\leq$  50 km intermediate: 50 km < d  $\leq$  300 km deep: d > 300 km

Ebru Bozdag



Dec. 19, 2009, Taiwan Mw=6.5, depth= 45.5 km Station: BOSA



# Conclusions

- "Adjoint tomography" is feasible
  - Goal: whole seismogram (frequency-dependent phase & amplitude)
  - Europe (Hejun Zhu)
  - Middle East (Daniel Peter)
  - Entire globe! (Ebru Bozdag)

	# earthquakes	# simulations	CPU core hours
Europe	160	11,200	806,400
Globe (Phase 1)	250	17,500	$14,\!437,\!400$
Globe (Phase 2)	5,000	350,000	739,200,000

- Future extensions:
  - Include amplitudes
  - Shear attenuation
  - More general anisotropy






































Fig. 1. Simplified tectonic map of Europe. TESZ, Trans-European Suture Zone.

## Tectonic map of Turkey







- Slab rollback western Mediterranean
  - I. Stage I: Algero-Provencal basin
  - 2. Stage 2: Tyrrhenian basin
- Eastern Mediterranean:
  - low heat flow, old (100 My), fast
- Western Mediterranean
  - high heat flow, young (25 My), slow
- Closing of Tethys, scissor with western hinge
- Hellenic arc old, cold, retreating (rollback)
- Cyprus arc, locked (no rollback)
- Cyprus hot spot: Erathostenes sea mount?
- Betic, Calabria and Vrancea subduction arcs are similar in appearance, with similar origins
- Adria subduction not seen previously
- Upper mantle hotspots not really seen
- Downdip tension in Vrancea: slab attached, with deposition basin above
- Downdip compression in Calabria: slab detached
- Alps due to Adia collision with Europe (Eocene-Oligocene)















## Early Cretaceous (~110 MYA)



## Late Cretaceous (~85 MYA)







## Middle Miocene (14 MYA)





